Design of Rogowski coil with integrator

Project on course:

High voltage generation and measurement

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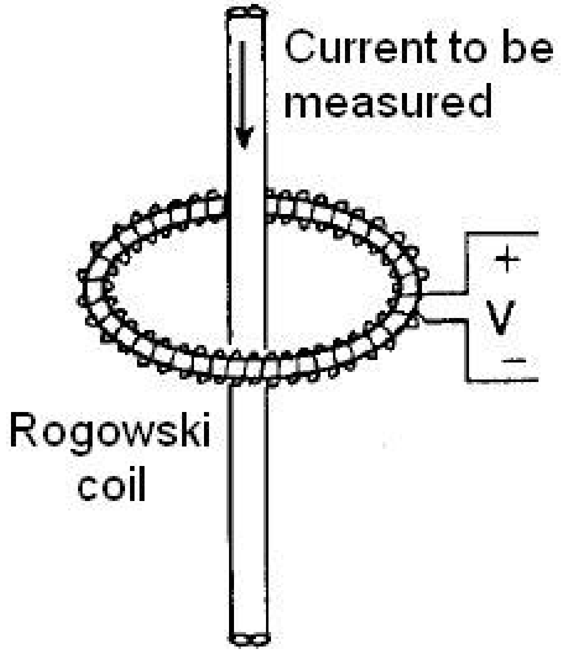
Design task and main advantages of the coil

Data:

I_{pk} = 200kA, risetime = 100ns,

low-frequency limit (3dB) 50Hz,

accuracy 2%, coil length > 0.5m

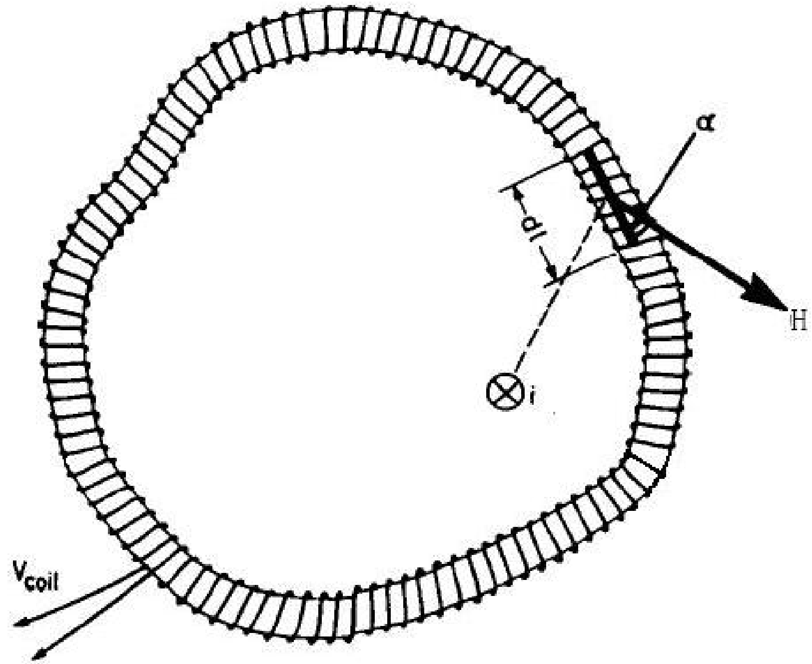


The advantages of using a Rogowski coil to measure high currents (having sharp rise) are:

- Linearity and non-saturation because it is air-cored.
- Simple circuitry, construction and low price.
- Non-intrusive, i.e., the coil does not load the circuit carrying the current to be measured under certain conditions of impedance matching.

The flux way independent output property

The coil is placed round the conductor and can be shaped in arbitrary form provided only that the ends of the coil are brought together. As showed below its output voltage is independent on the magnetic flux way caused by a measured current.



$$i = \oint H \cos\alpha \cdot dl \quad M = \mu_0 n A$$

$$d\Phi = M \cdot H\, dl \cos\alpha \quad \Phi = \int d\Phi$$

$$\Phi = M \int H \cos\alpha dl = M \cdot i$$

$$V_{coil} = -\frac{d\Phi}{dt} = -M\frac{di}{dt};$$

Variety of the Rogowski coil windings

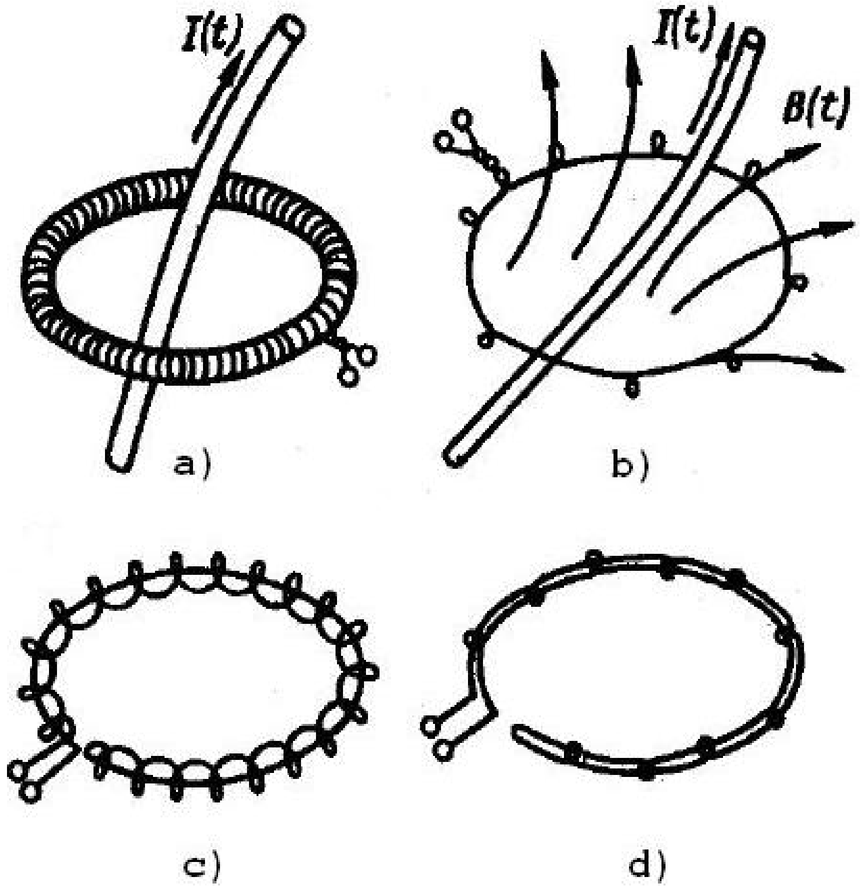
On the right depicted:

a) toroidal Rogowski coil

b) turn formatted by the coil

c) coil with compensating turn

d) coil with opposite windings



l – length of middle line

S – square (cross section)

w – number of turns

Electromotive force induced in the coil:

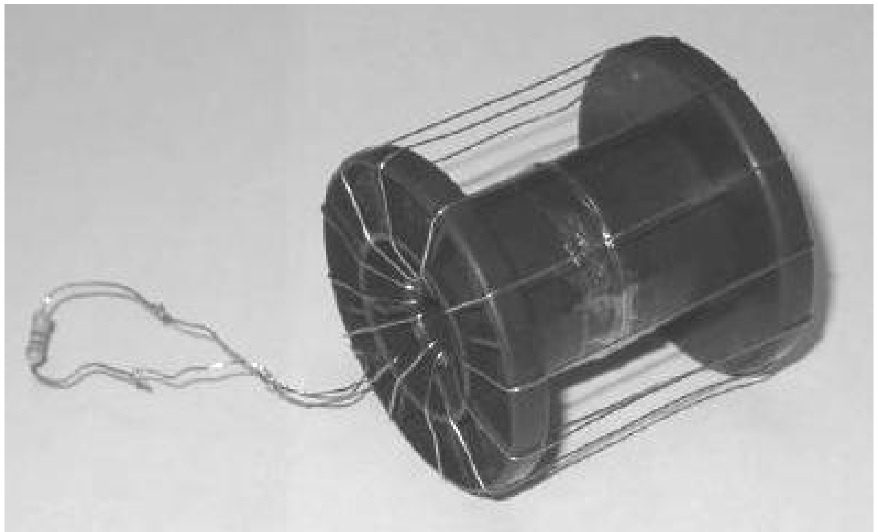
$$\varepsilon(t) = -\frac{d}{dt}\oint_{\ell} \mu_0 \frac{w}{l} dl \iint_S H dS = -\mu_0 \frac{w}{l} S \frac{d}{dt}\oint_{\ell} H dl = -\mu_0 \frac{w}{l} S \frac{dI}{dt}$$

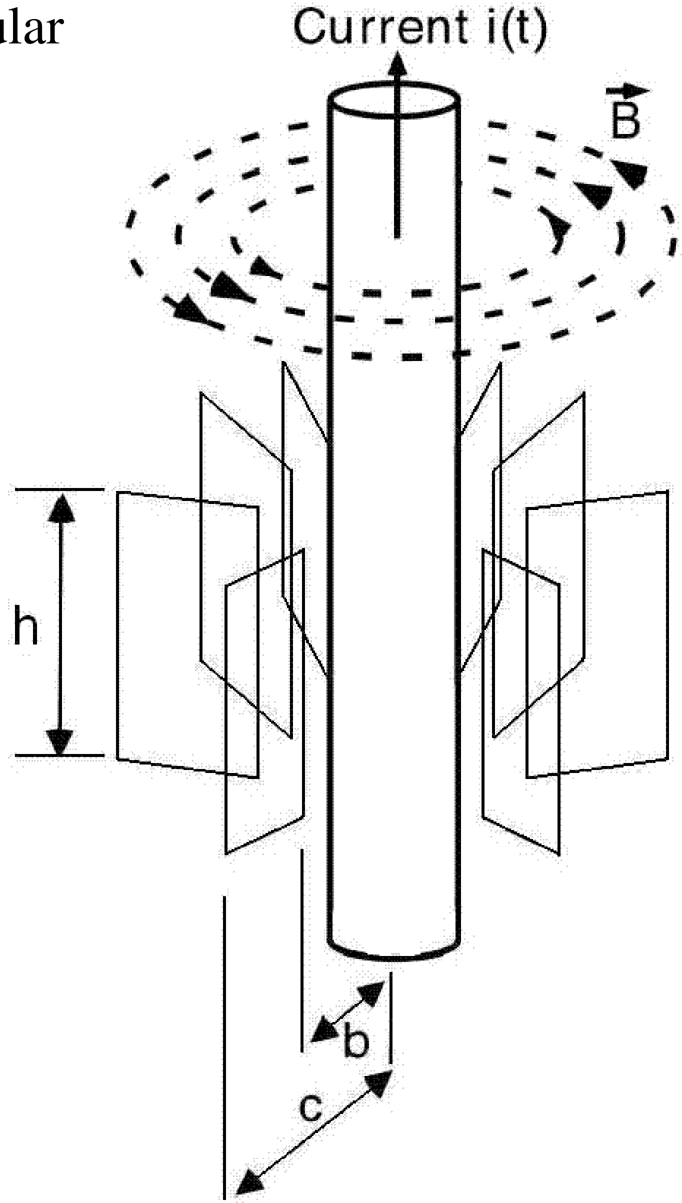
The coil with rectangular winding

Electromotive force outputs from a coil with rectangular air core around a long straight wire is:

$$\varepsilon(t) = \frac{\mu_{air} \cdot N \cdot h}{2 \cdot \pi} \cdot \ln\left(\frac{c}{b}\right) \cdot \frac{di(t)}{dt}$$

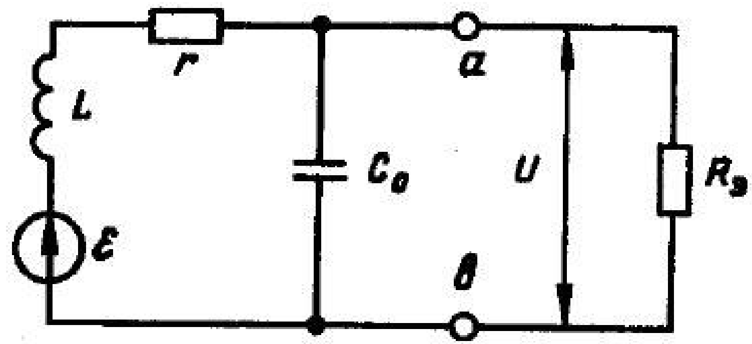
Its inductance is: $L = \frac{\mu \cdot N^2 h}{2\pi} \ln\left(\frac{c}{b}\right)$





Equivalent circuit of the Rogowski coil

The transfer function of the equivalent circuit is defined as:



$$W(s) = \frac{K \cdot s}{T^2 s^2 + 2 \cdot \xi \cdot T \cdot s + 1}$$

$$\xi = \frac{L + r \cdot C_0 \cdot R_{eq}}{2\sqrt{L \cdot C_0 \cdot R_{eq} \cdot (r + R_{eq})}}$$

$$K = \frac{\mu_{ra} \cdot S_k \cdot K_{div}}{r + R_{eq}}$$

$$\xi(R_{eq}) \to \infty \text{ when } R_{eq} \to 0$$

$$T = \sqrt{\frac{L \cdot C_0 \cdot R_{eq}}{r + R_{eq}}}$$

$$\xi(R_{eq}) \to \frac{r}{2}\sqrt{\frac{C_0}{L}} \text{ when } R_{eq} \to \infty$$

The transfer function has very different features when $0 < \xi < 1$, $\xi \approx 1$, $\xi >> 1$

The case of high current measurement

The parameter ξ in the transfer function can be represented in other form:

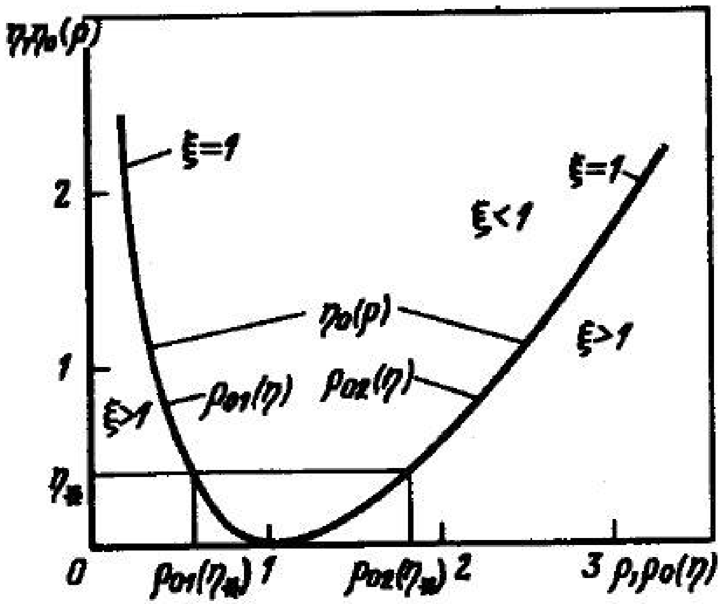
$$\xi(\rho,\eta)=\frac{\rho+\rho^{-1}}{2\sqrt{1+\eta}};\ \rho=\sqrt{\frac{L}{r\cdot R_{eq}C_0}};\ \eta=\frac{R_{eq}}{r}$$

When a high current is measured a load with high resistance is using to reduce the coil current.

In this case the condition $\xi >> 1$ takes place if

$$\rho=\frac{1}{R_{eq}}\sqrt{\frac{\eta'\cdot L}{C_0}} << \sqrt{1+\eta'}-\sqrt{\eta'}=\rho_{01}(\eta')<1$$

An additional, big enough capacitance in parallel with load resistor is necessary.



relation between ρ and η when ξ(ρ,η)=1

$$\eta_0(\rho)=\left(\frac{\rho-\rho^{-1}}{2}\right)^2$$

$$\rho_{01}(\eta)=\sqrt{\eta+1}-\sqrt{\eta}$$

$$\rho_{02}(\eta)=\sqrt{\eta+1}+\sqrt{\eta}$$

Critical parameters at high resistance load

The transfer function (sensitivity) is:

$$W_0 \approx \frac{K_{div} \cdot R_{eq}}{w} \rho^2 << \frac{K_{div} \cdot R_{eq}}{w}$$

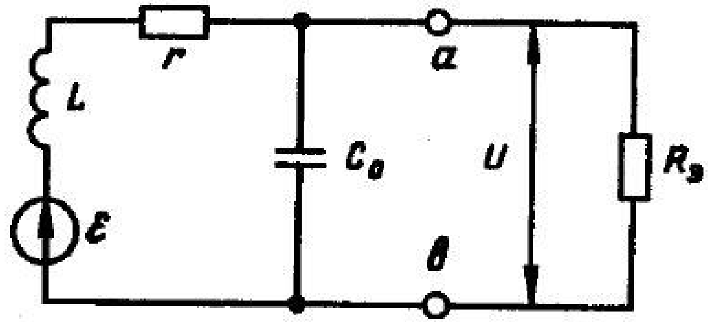
Maximal current in the coil turns:

$$I_{\max}^c = \frac{W_0 \cdot I_{\max}\left(t_p + R_{eq} \cdot C_0\right)}{K_{div} \cdot t_p \cdot R_{eq}}$$

$$I_{\max}^c \approx \frac{I_{\max}}{w}\left(1 + \frac{R_{eq} C_0}{t_r}\right) \cdot \rho^2$$

Low frequency limit is:

$$f_{dw} = \frac{1 + \eta'}{2\pi \cdot R_{eq} C_0}$$



High frequency limit is:

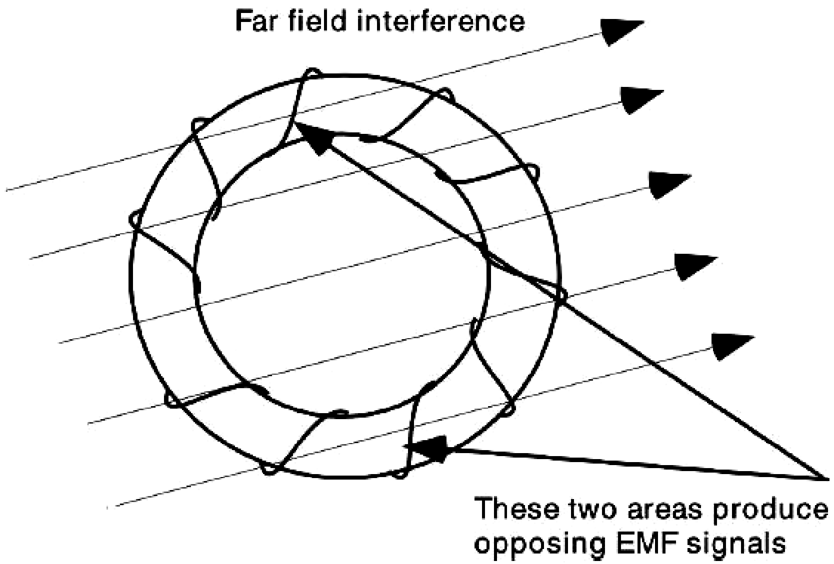
$$f_{up} = \frac{\rho + \rho^{-1}}{2\pi \cdot \sqrt{\eta \cdot LC_0}} \approx \frac{r}{2\pi \cdot L}$$

(*) When measured current has relatively sharp rise it is limited also by total wire length and light velocity:

$$f_{up} = \frac{c}{10 \cdot l_{\Sigma\, wire}}$$

Cancellation of external interferences

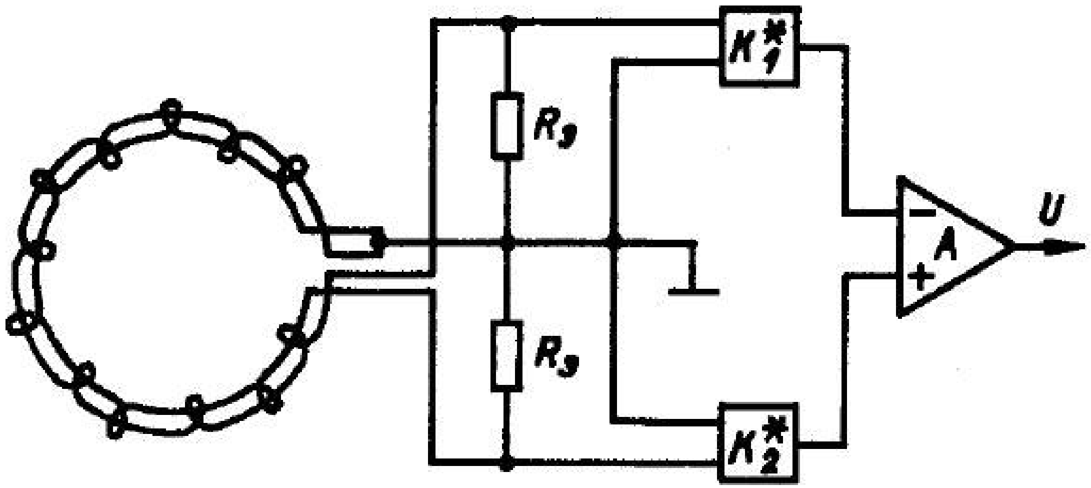
- Interference is usually "far field" and common mode signal in nature
- Conductor loop will pick up both signals and interference
- Shielding low frequency magnetic field is difficult and requires thick shielding or high permeability material
- Minimizing unwanted loop and far field interference cancellation will provide much more benefits than shielding along

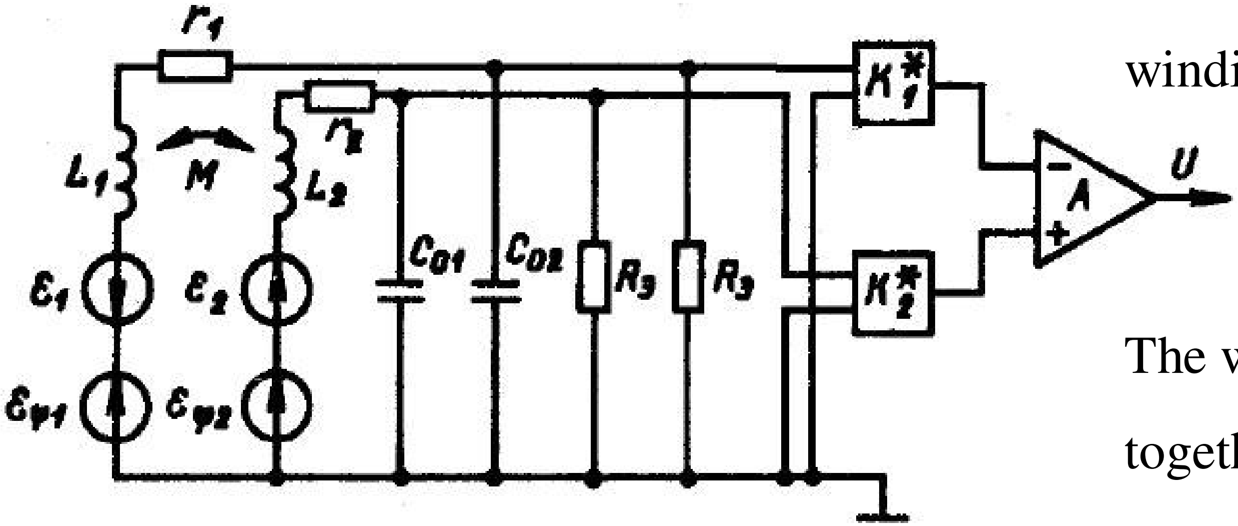


(*) The voltage of each turn is synchronous and hence rise time of the output signal can be significantly less than wave run time on the winding. This fact define a high frequency limit by light velocity and total wire length:

$$f_{up} = \frac{c}{10 \cdot l_{\Sigma\, wire}}$$

Differential coil with opposite winding



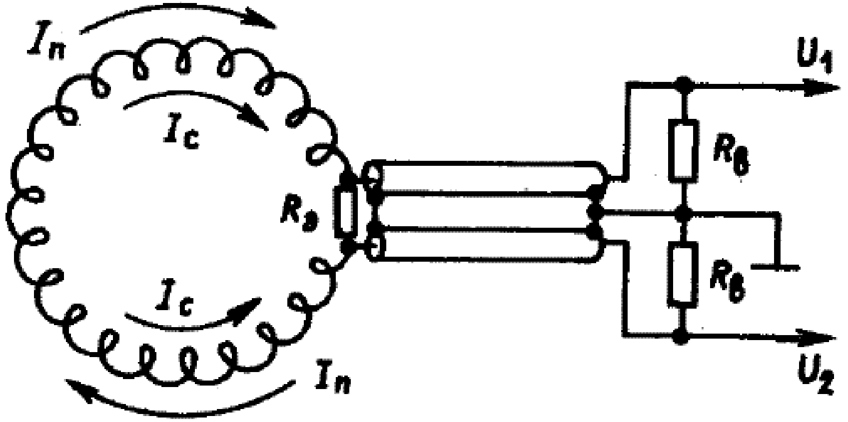


The coil include unwanted loop that can sense outside magnetic flux.

If its spectrum lie although partly in bandwidth of measured current an additional error is arising. One of the methods to reduce this error is using winding with opposite turns.

The winding is depicted on the left together with its equivalent circuit.

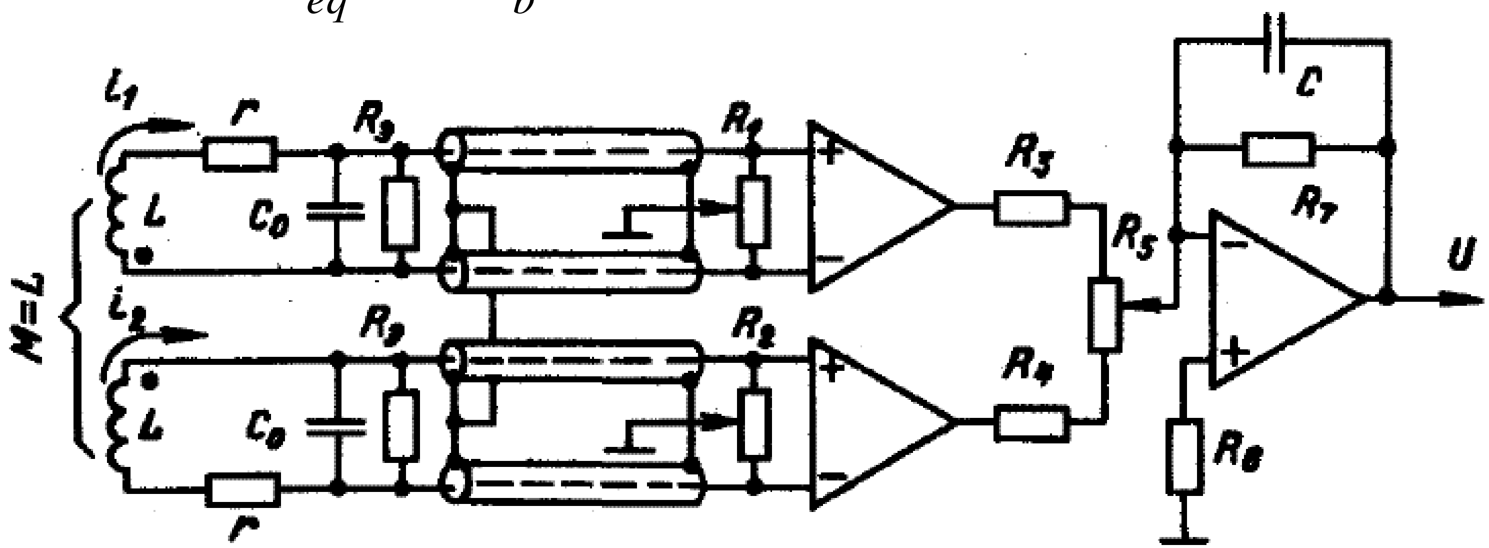
Getting rid from percussive excitation



A common mode current I_c cause percussive excitation in the output coil voltage. In case of symmetrical load I_c is cancelled and just useful differential current I_n takes place.

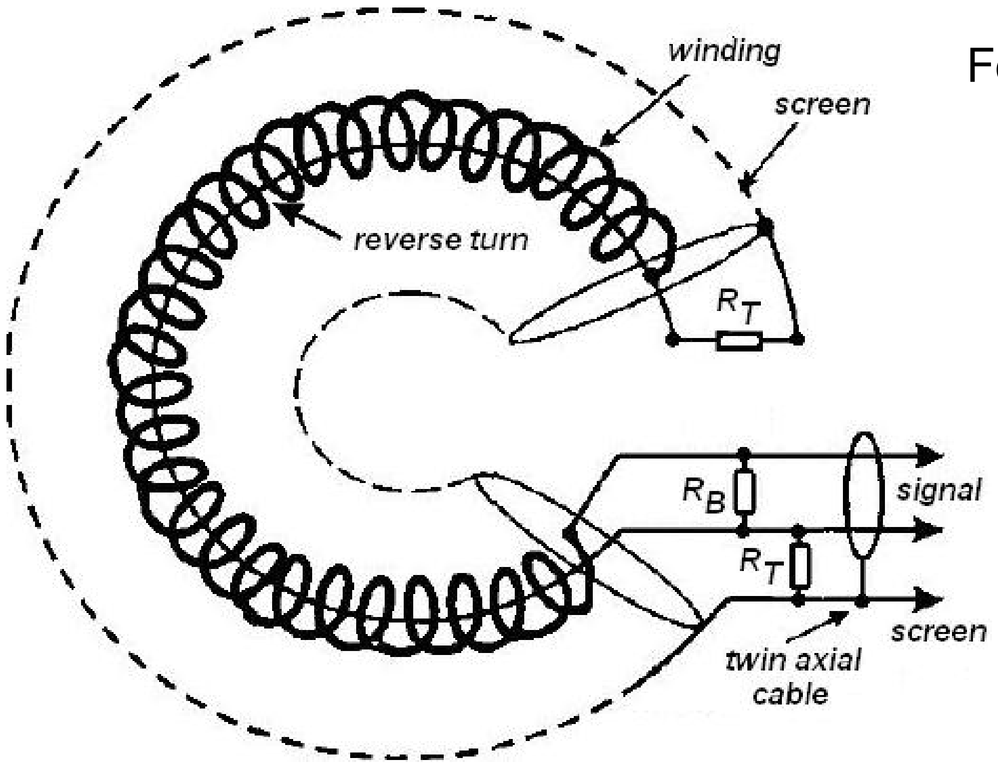
$$V = V_1 - V_2 = I_n \frac{2 \cdot R_{eq} \cdot R_b}{R_{eq} + 2 \cdot R_b}$$

Using of two separated opposite winding coils is shown below.



Shielding arrangement for the coil

The response of the copper shielded coil contains a large high frequency oscillations due to the screen and coil acting together as a coaxial transmission line. This oscillation component can be removed by terminating both ends in the characteristic impedance Z_T.



For a concentric circular coaxial line in air:

$$Z_T = 138 \cdot \log \frac{D}{d}$$

D is the inner diameter of the screen

d is the outer diameter of the coil

Integration methods and self –integrating coil

- Electronic passive or active (using op-amp) integration networks.
- Digital integration with DSP or data acquisition card, characterized by:
  - Stability of performance due to absence of the drift over time and temperature
  - Outstanding phase features without slight variation and not requiring calibration
  - Relatively low cost because no precision external component is needed
- Self-integration by specially designed coil, as it is explained below.

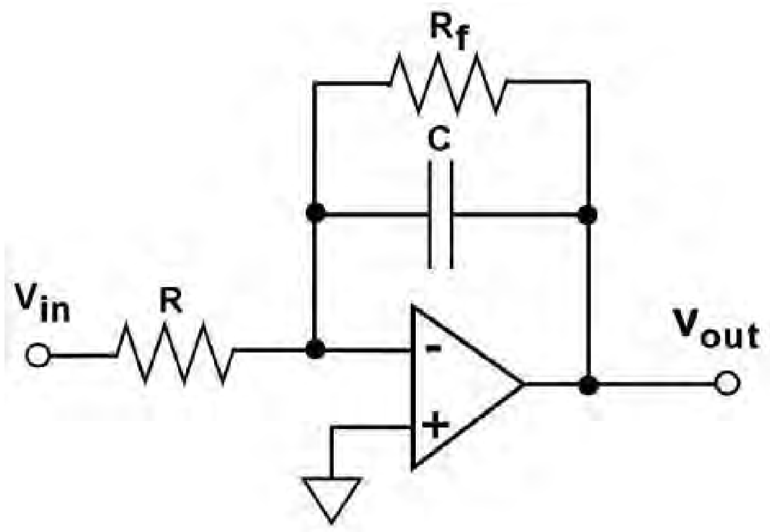
A self-integrating Rogowski coil is based on using a low enough resistance as terminating impedance *Z* so that the transfer function for sinusoidal current:

$$\frac{V_{out}}{V_{coil}} = \frac{R_{out} \parallel (1/s \cdot C_c)}{R_c + s \cdot L_c} = \frac{R_{out}}{R_c + s \cdot L_c}$$ for high frequency current $R_C << s \cdot L_S$ and

$$V_{out} = \frac{R_{out}}{s \cdot L_c} \cdot V_{coil} = \frac{R_{out}}{L_c} \cdot M \cdot i = k \cdot i$$ where $k = \frac{R_{out}}{L_c} \cdot M$ is the coil sensitivity

Simplest inverting integrator using an op-amp

Integration in time domain is equivalent to the following in the frequency domain:



`Magnitude:` -20dB/decade

`Phase:` 90° phase shift for all frequency

Assuming $R_f >> R$

in time domain:

$$V_{out}(t) = -\frac{1}{RC}\int V_{in}(t)\,dt$$

in frequency domain:

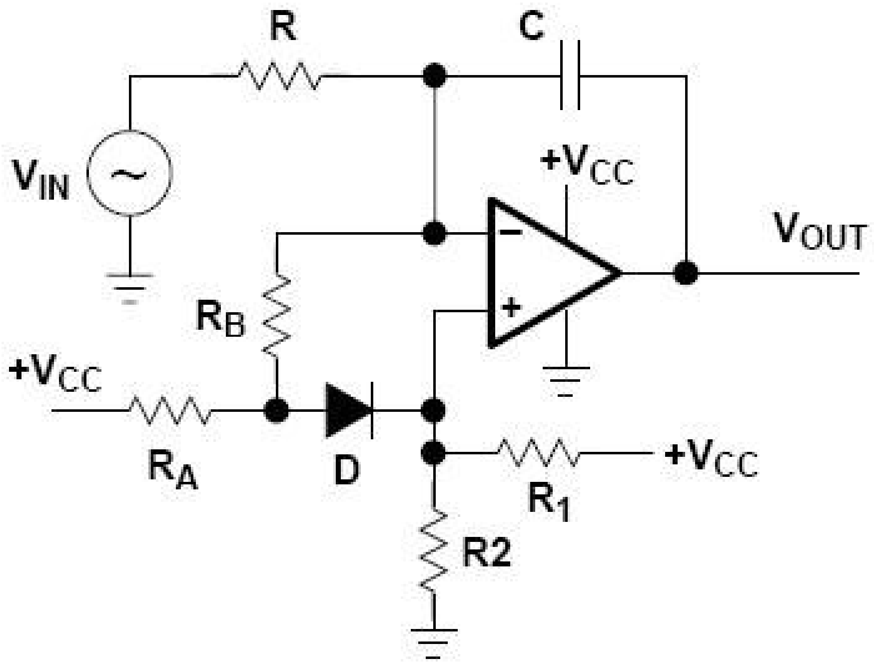
$$V_{out} = -\frac{V_{in}}{s \cdot RC}; \quad s = j\omega$$

<u>Error includes:</u>

offset error, leakage current, long-term stability of op-amp and temperature drift

Integrator with input current compensation

In order to bias forward the diode R_1 in parallel with R_2 are relatively small. If the diode current is selected correctly both diode and op-amp input transistors temperature sensitivities cancel out. The resistor R_B is large, so the op-amp is biased and no input current flows through the integration resistor R.

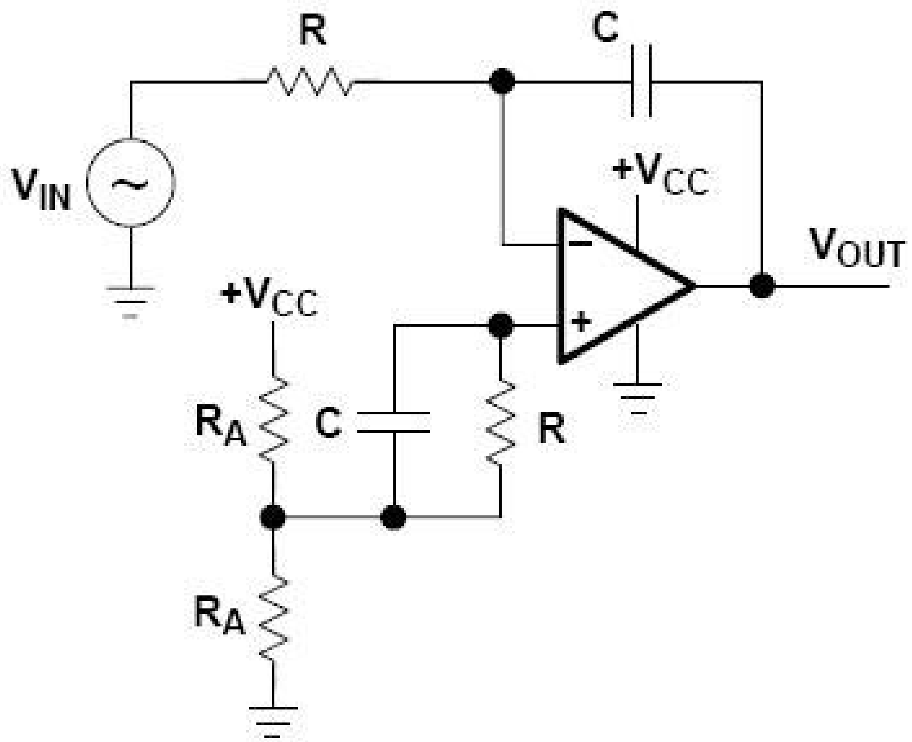


This integrator is not very practical because there is no capacitor discharging.

This bias circuit is intended for an op-amp that has NPN input transistors. The diode must be reversed and connected to ground for op-amps with PNP input.

Inverting integrator with drift compensation

The positive input current drops the same voltage across the parallel RC combination as the negative input current drops across its series RC combination.

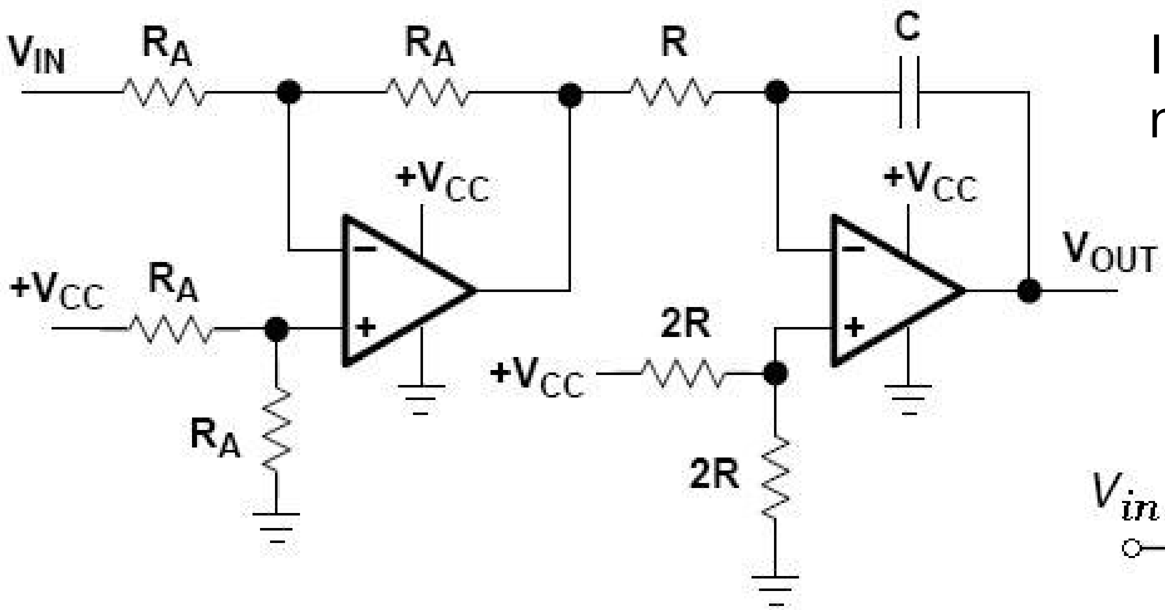


The common-mode rejection capability of the op-amp rejects the voltages caused by the input currents.

Much longer integration times can be achieved with this circuit, but when the input signal does not center around $V_{CC}/2$, the compensation is poor.

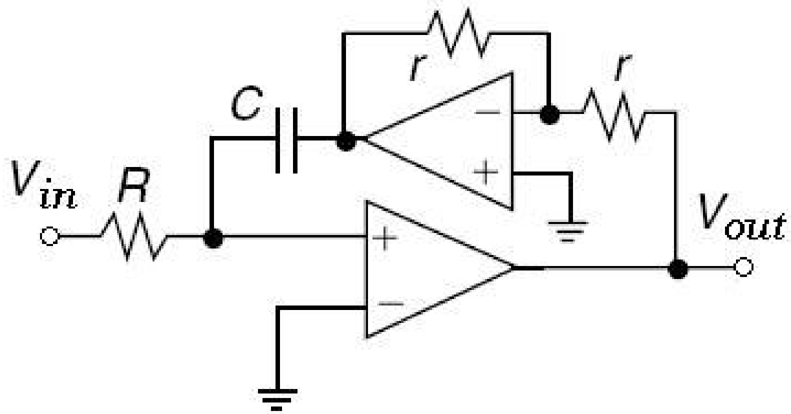
Non-inverting integrator with inverting buffer

As known double inversion is positive. The same lossless integrator can be obtained by addition of inverting buffer with absolute gain slightly more than one. This easiest way to get true non-inverting performance costs just one more op-amp.



Input buffer lossless non-inverting integrator

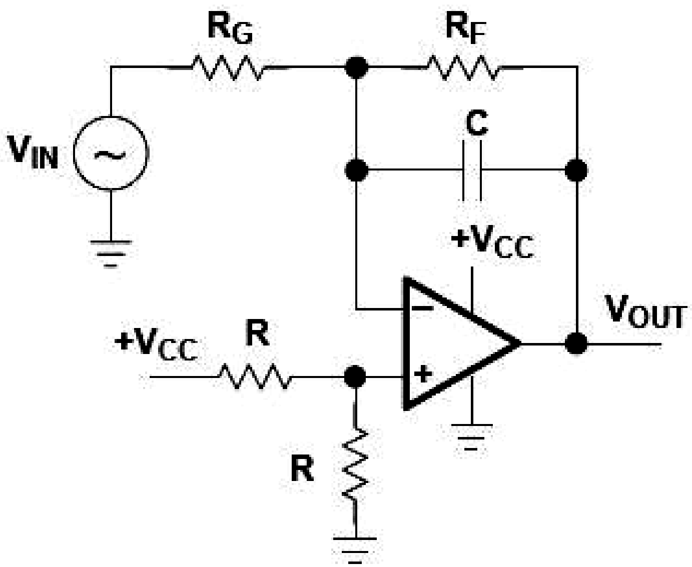
$$V_{out} = -\frac{R_A}{R_A}\left(-\frac{V_{in}}{s \cdot RC}\right) = \frac{V_{in}}{s \cdot RC}$$



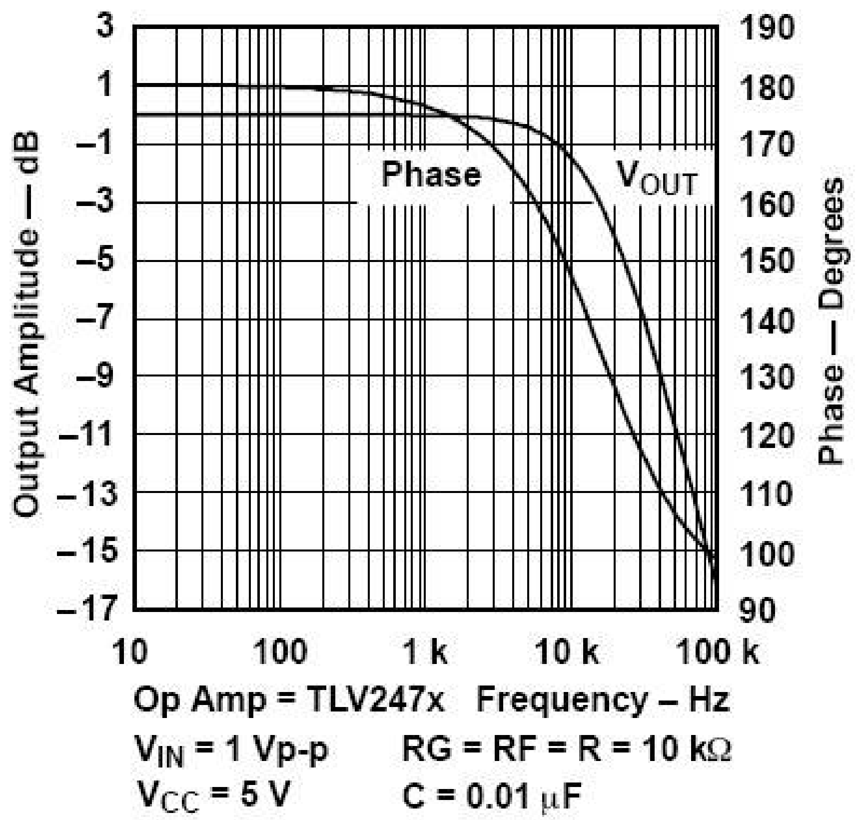
Output buffer lossless non-inverting integrator

Inverting integrator with resistive reset

In this circuit the low frequency attenuation is sacrificed for the reset circuit. There is continuous discharging of the integrating capacitor. A breakpoint plots flat until the breakpoint where it breaks down at –6 dB per octave. It is –3 dB when $f = 1/2\pi RC$.

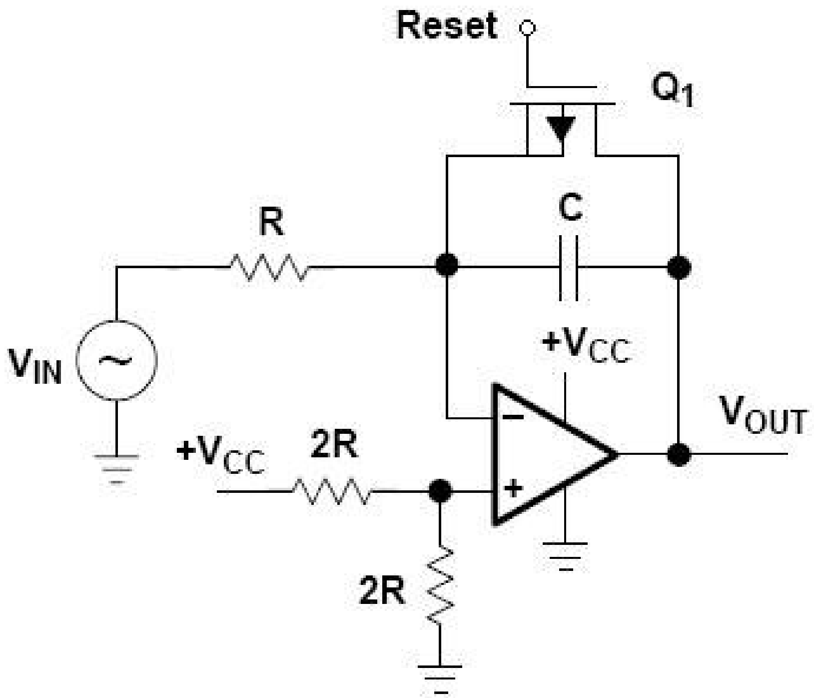


$$V_{out} = -\frac{V_{in}}{R_F C + 1} \cdot \frac{R_F}{R_G}$$



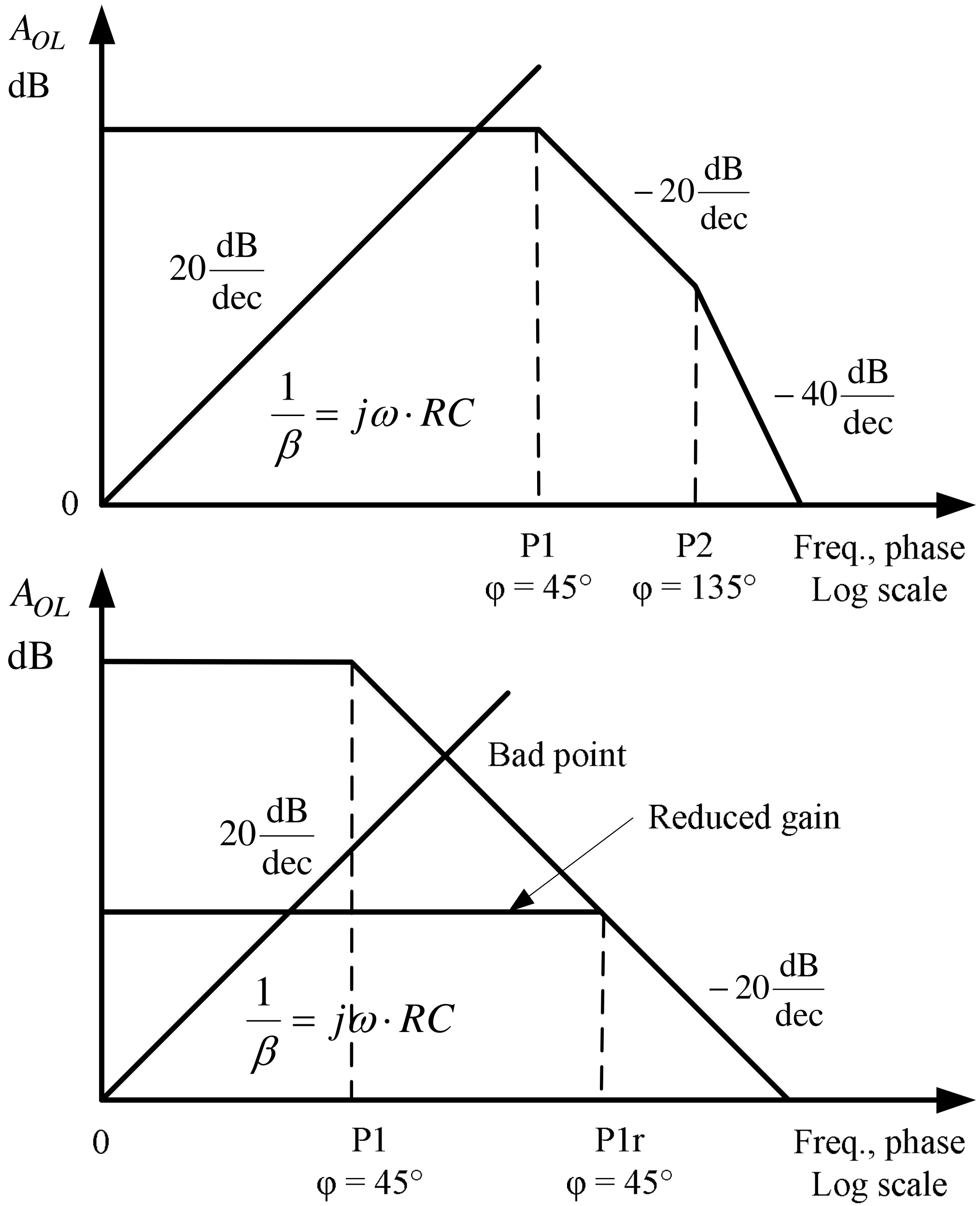
Inverting integrator with electronic reset

The source of the FET is at the inverting lead that is at ground, so the Q1 gate source bias is not affected by the input signal. Sometimes, the output signal can get large enough to cause leakage currents in Q1, so the designer must take care to bias Q1 correctly.

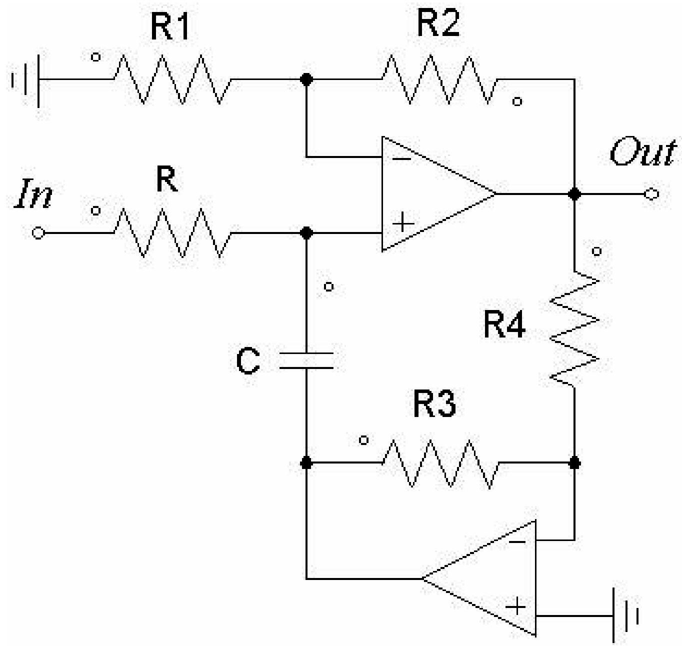


A major problem with electronic reset is the charge injected through the transistor's stray capacitance. This charge can be large enough to cause integration errors.

Stability of the op-amp based integrator

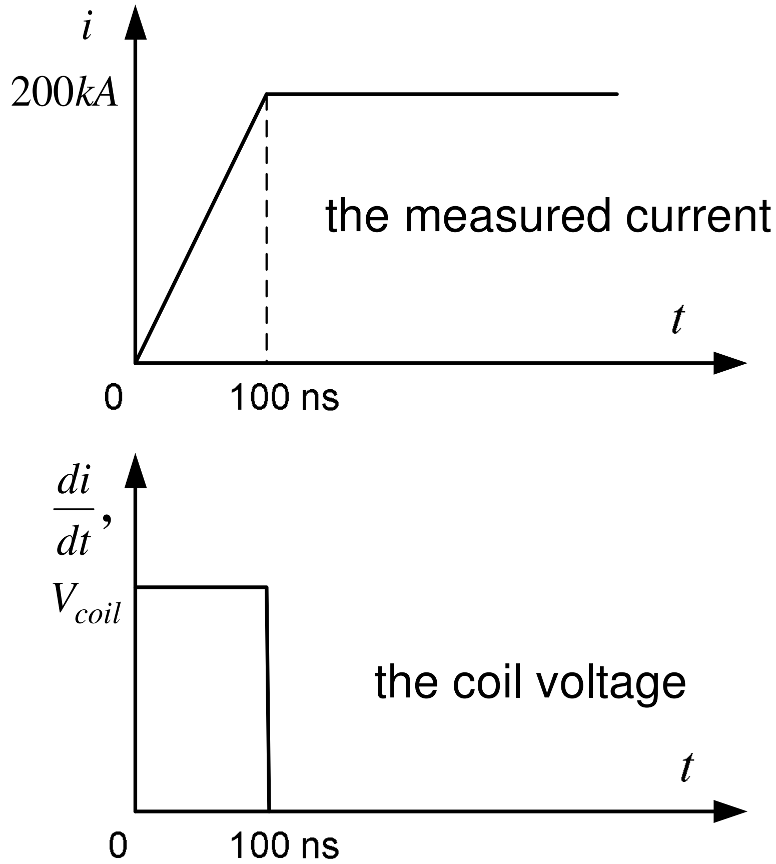


In order to get the right cross point a reduction of the op-amp gain according to its frequency response and phase margin may be necessary.



Considerations for given task solution

It is obvious that after differentiation of the current rise an rectangular pulse of the coil voltage is obtained, its width is the same rise time.



Because an low-cost integrator is desired it should be passive.

In case of cheap active integrator the op-amp slew rate should be medial relatively.

Assume $200kA$ is corresponding to $1V$ output voltage of the integrator.

The slew rate in this case $1V/100ns$, i.e. SR = $10V/us$ that is OK.

Preliminary calculation for square coil

Data and values chosen in first iteration

$\mu_{air} := 1.26 \cdot 10^{-6}$ $\quad l_{min} := 0.5$ $\quad I_{max} := 200 \cdot 10^{3}$

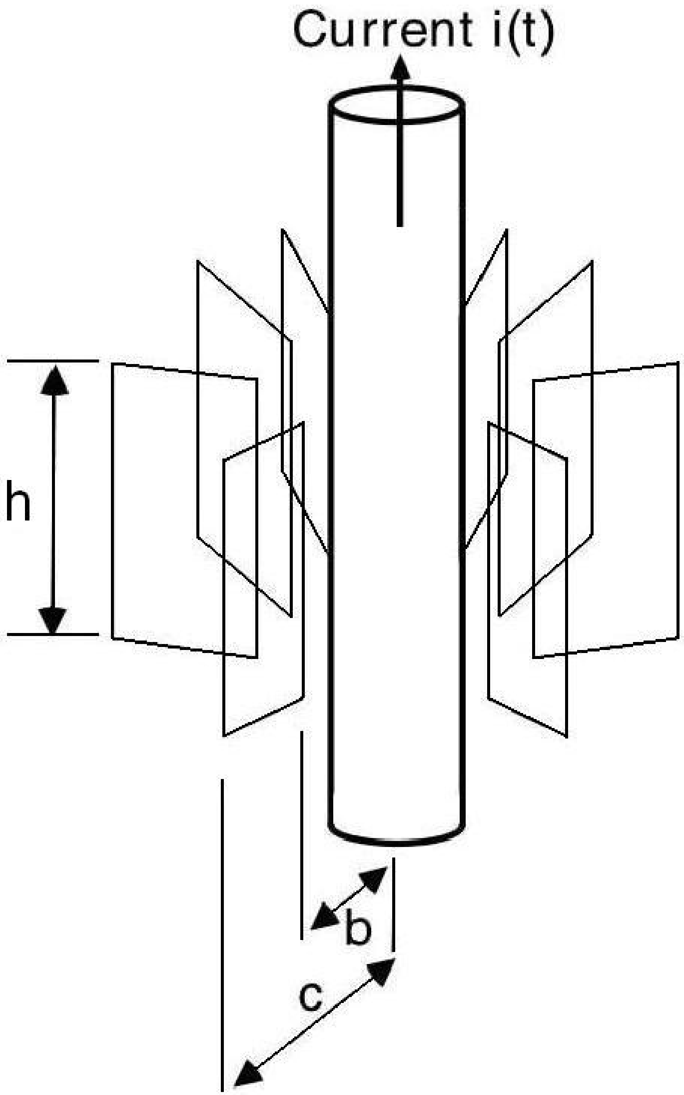
$\tau := 100 \cdot 10^{-9}$ $\quad r := 1 \cdot 10^{6}$ $\quad R_{eq} := \frac{r}{100}$

$\delta := \frac{I_{max}}{\tau}$ $\quad \delta = 2 \times 10^{12}$ $\quad V_{coil} := 3000$

$h := 0.002$ $\quad b := \frac{l_{min}}{2\pi}$ $\quad$ Let $\quad c := b + h$

$b = 0.08$ $\quad c = 0.082$ $\quad S := h^{2}$ $\quad S = 4 \times 10^{-6}$

$\eta := \frac{R_{eq}}{r}$ $\quad \eta = 0.01$ $\quad \lambda := ln\left(\frac{c}{b}\right)$ $\quad \lambda = 0.025$



Results and simulation with integrator

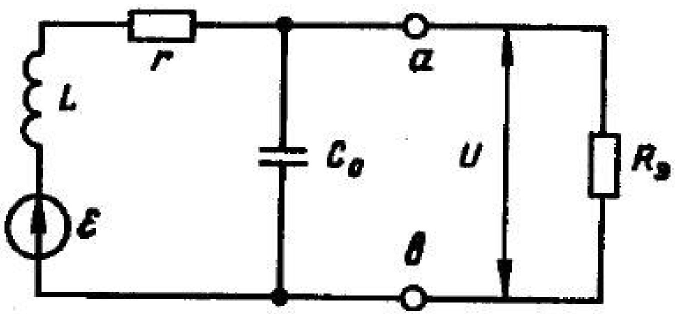
for rectangular coil: $N_r := \frac{2\pi \cdot V_{coil}}{\mu_{air} \cdot \lambda \cdot \delta \cdot h}$ $N_r = 150.672$ $L_r := \frac{\mu_{air} \cdot N_r^2 \cdot h}{2 \cdot \pi} \lambda$ $L_r = 2.26 \times 10^{-7}$

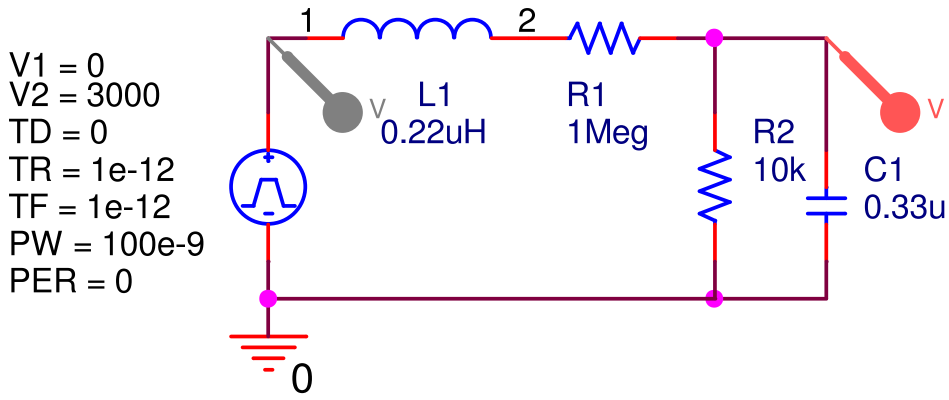
for toroidal thin coil: $N_t := \frac{V_{coil} \cdot l_{min}}{\mu_{air} \cdot S \cdot \delta}$ $N_t = 148.81$ $L_t := \frac{\mu_{air} \cdot N_t^2 \cdot S}{\pi \cdot (b + c)}$ $L_t = 2.204 \times 10^{-7}$

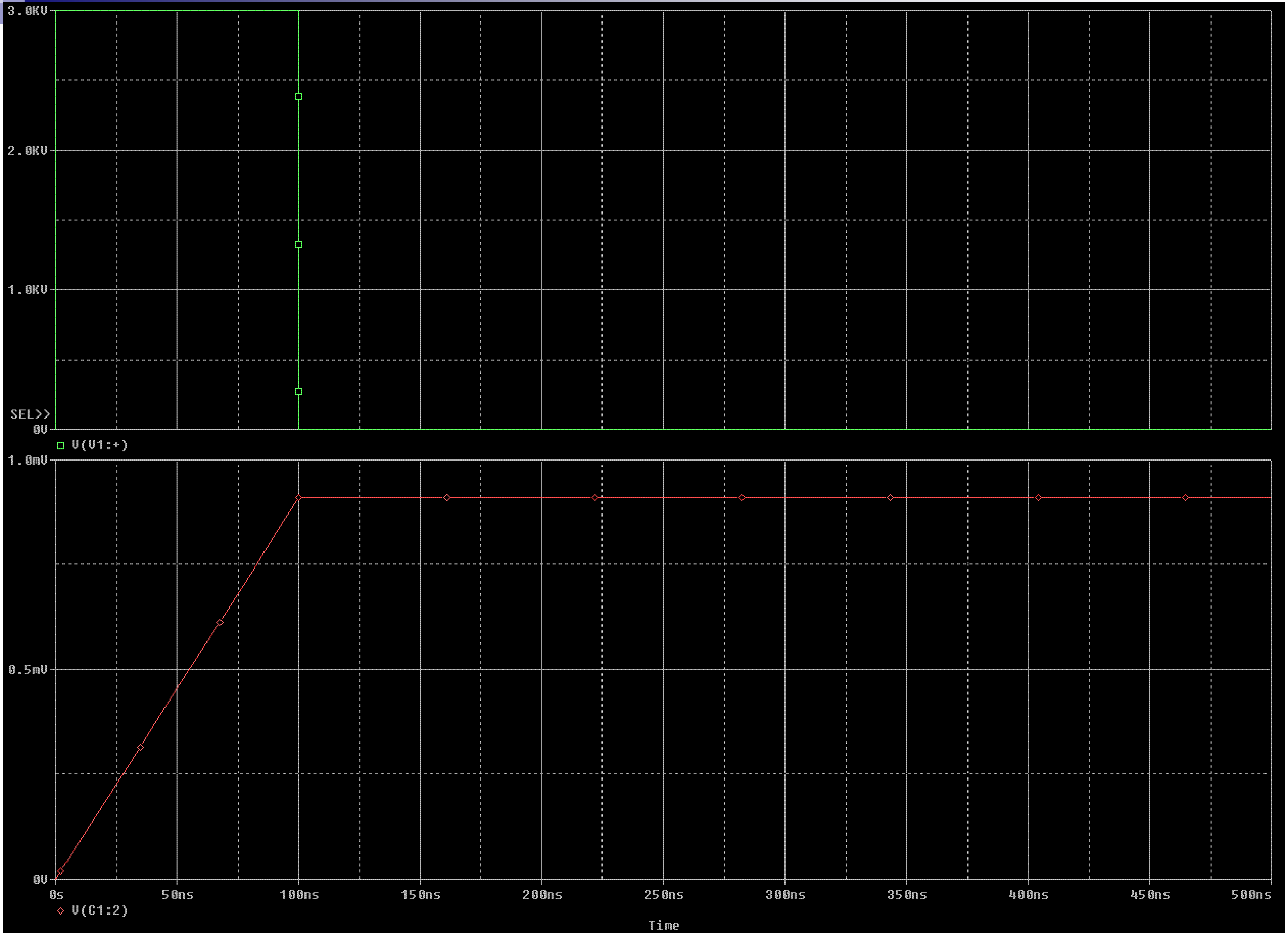
Let $C_0 := 0.33 \cdot 10^{-6}$ then $\rho := \sqrt{\frac{L_t}{r \cdot R_{eq} \cdot C_0}}$ $\rho = 8.173 \times 10^{-6}$ $\rho^2 = 6.68 \times 10^{-11}$

Transfer function (mV/kA): $W_0 := \frac{R_{eq}}{N_r} \cdot \rho^2 \cdot 10^6$ $W_0 = 4.434 \times 10^{-3}$ $W_0 \cdot 200 = 0.887$

So for maximal measured current 200 kA output voltage is 887 uV







3.0KV
2.0KV
1.0KV
SEL>>
0V
V(V1:+)
1.0mV
0.5mV
0V
0s
50ns
100ns
150ns
200ns
250ns
300ns
350ns
400ns
450ns
500ns
V(C1:2)
Time

Calculation with circular polymer core

The inner polymer vein of the cable RG-58 can be used as core of the coil. This dielectric has diameter 2.95 mm and made from gas-injected foam high density polyethylene.

Cable RG-58

$d := 2.95\cdot 10^{-3}$ $\quad S := \pi\cdot\left(\frac{d}{2}\right)^2$ $\quad \sqrt{S} = 2.614\times 10^{-3}$ $\quad l := l_{min} + \pi\cdot d$ $\quad l = 0.509$

$N := 88$ It is fair to assume that magnetic permeability of polyethylene μ_{pol} is approximately μ_{air} , in other words: $\mu_{pol} := 1.26\cdot 10^{-6}$

$V_{coil} := \mu_{pol}\cdot\frac{N\cdot S}{l}\cdot\delta$ $\quad V_{coil} = 2.976\times 10^{3}$ $\quad L := \mu_{pol}\cdot\frac{N^2\cdot S}{l}$ $\quad L = 1.31\times 10^{-7}$

$r := 1\cdot 10^{6}$ $\quad R_{eq} := \frac{r}{2}$ Let $C_0 := 10\cdot 10^{-9}$ then $\rho := \sqrt{\frac{L}{r\cdot R_{eq}\cdot C_0}}$

Transfer function (mV/kA): $W_0 := \frac{R_{eq}}{N}\cdot\rho^2\cdot 10^{6}$ $\quad W_0 = 0.149$

So for maximal measured current 200 kA output voltage is 29.763 mV

Calculation with another more thick core

As known the cable RG-59 is more thick than RG-58. The diameter of the RG-59 inner dielectric is 3.71 mm, so number of the coil turns will be smaller.

$N := 56 \quad \sqrt{S} = 3.288 \times 10^{-3} \quad l = 0.512 \quad V_{coil} = 2.982 \times 10^{3} \quad L = 8.348 \times 10^{-8}$

$\rho = 4.086 \times 10^{-6}$ Transfer function (mV/kA): $W_0 = 0.149 \quad W_0 \cdot 200 = 29.816$

So for maximal measured current 200 kA output voltage is 29.816 mV

$\eta := \frac{R_{eq}}{r} \quad f_{dw} := \frac{1+\eta}{2\cdot\pi\cdot R_{eq}\cdot C_0} \quad f_{dw} = 47.746 \quad f_{up} := \frac{r}{2\cdot\pi\cdot L} \quad f_{up} = 1.906 \times 10^{12}$

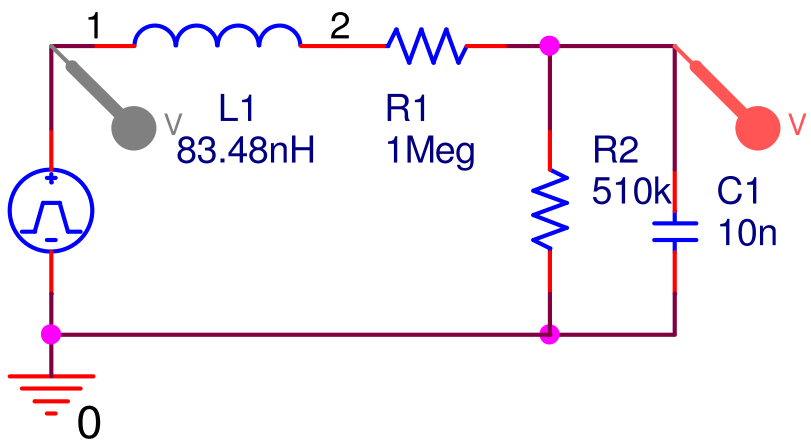
Here the high frequency limit is difined by the wire length and light velocity:

$x := \pi\cdot d\cdot N + l \quad x = 1.164 \quad lv := 3\cdot 10^{8} \quad f_{up} := \frac{lv}{10\cdot x} \quad f_{up} = 2.577 \times 10^{7}$

Maximal current in the coil: $I_c := \frac{W_0}{10^6}\cdot\left(\frac{1}{R_{eq}} + \frac{C_0}{\tau}\right)\cdot I_{max} \quad I_c = 2.982 \times 10^{-3}$

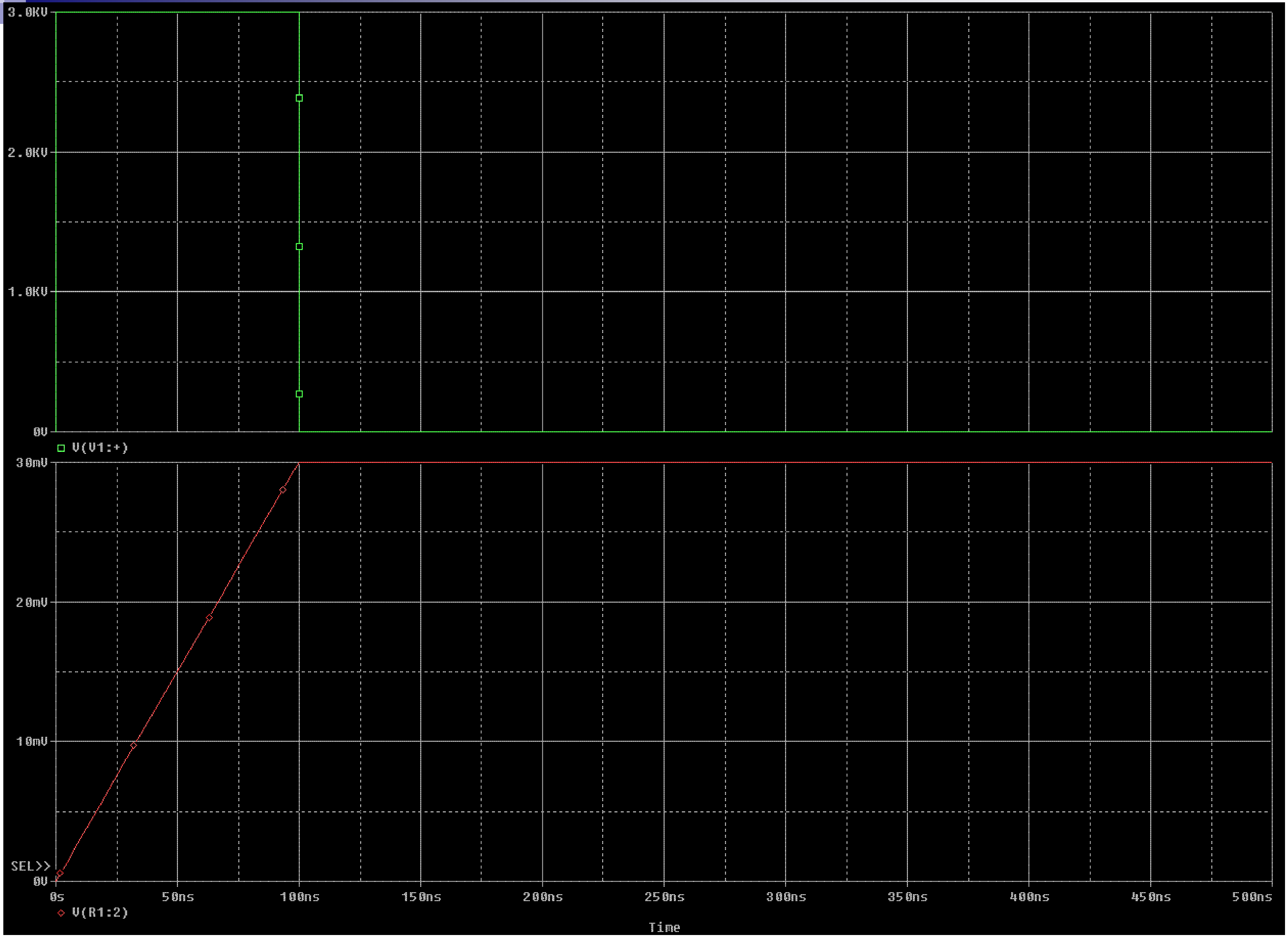
Processing of RG-58 cable:

1) Remove the outer isolation
2) Pull off the wicker screen
3) Gather the glued foil tape
4) Wind the necessary turns
5) Shape the circle form coil

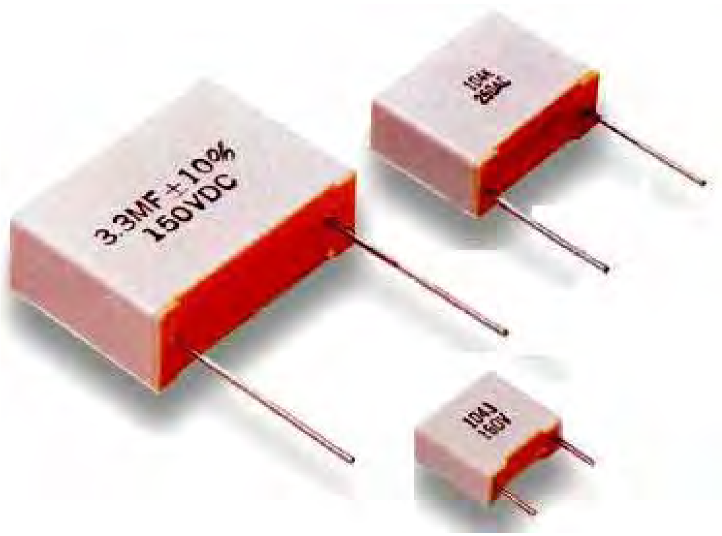


The winding is 88 turns of the wire with teflon isolation that has diameter 0.8 mm.

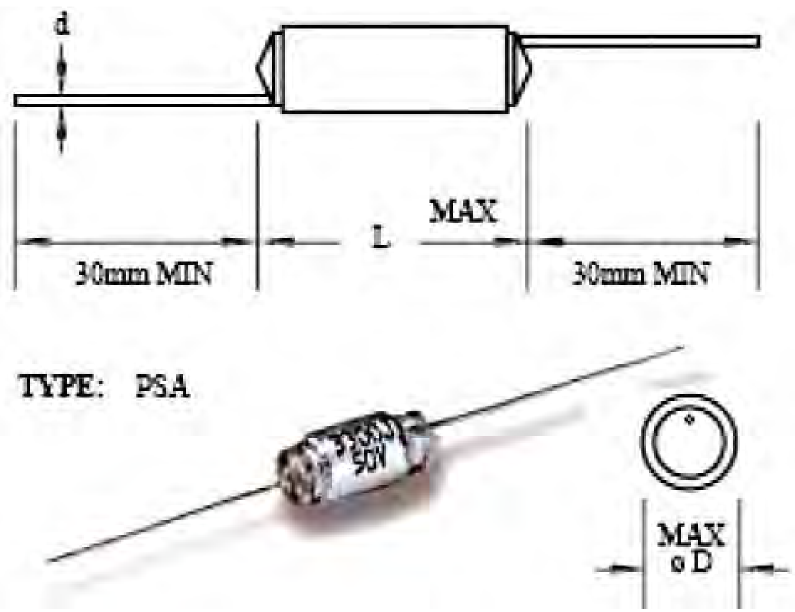
The central conductor is not connected in loop, its isolated ends just brought together.


3.0KV
2.0KV
1.0KV
0V
V(V1:+)
30mV
20mV
10mV
SEL>>
0V
0s
50ns
100ns
150ns
200ns
250ns
300ns
350ns
400ns
450ns
500ns
V(R1:2)
Time

Choosing of specific integrator capacitor



It is revealed that capacitors have worse precision than resistors, hence discuss this first. Relatively cheap capacitors of 10 nF have best tolerance 1%. They was found at Suntan Hong Kong. There are two models TS02C - metallized polycarbonate film (above) and TS10 - polystyrene film capacitors (below) .



Main features:

- Low dissipation factor and leakage
- High stability versus frequency
- Temperature coefficient 100 ppm/℃

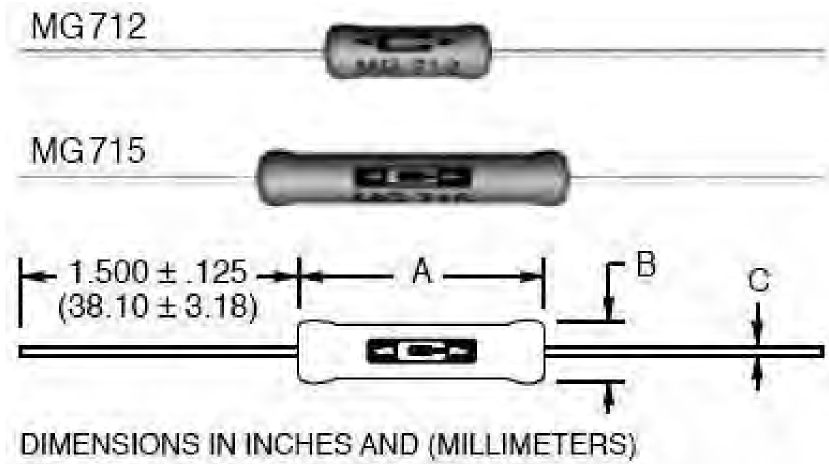
VDC / PF	50VDC/100VDC			160VDC			630VDC		
	D	L	Ø d	D	L	Ø d	D	L	Ø d
10000	10	15	0.4	10.5	22	0.5	13	24	0.5

Choosing of specific integrator resistors

Among host models of high voltage resistors one of the most suitable is MG7 series produced by Caddock Electronics, Inc. Because short duration of the current pulse due resistor power is low comparing with DC 3 kV×2.98 mA case. The chosen parameters given in the table. If total accuracy 2%, capacitor tolerance 1% then precision of each 1M and 510k resistor is better 0.5%.

Main features:

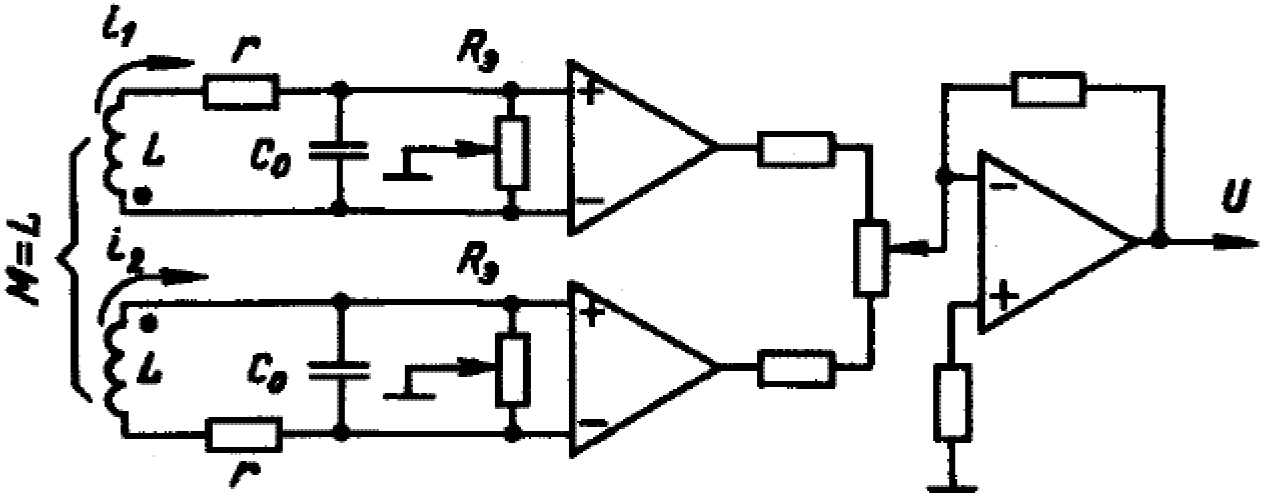
- Overvoltage capabilities of 150%
- Tolerances from ±1.0% to ±0.1%.
- Temperature coefficient 80 ppm/℃



Model No.	Watt-age	Max. Continuous Oper. Volt.	Overload Rating	Dielect. Strength	Resistance			Dimensions in inches and (millimeters)		
					Min.	-15 Min.	Standard Max.	A	B	C
MG712	0.6	1,000	Type 2	750	800 Ω	N/A	20 Meg	.400 ±.060 (10.16 ±.152)	.140 ±.030 (3.56 ±.76)	.025 ±.002 (.64 ±.05)
MG715	1.0	2,000	Type 2	750	400 Ω	26 Meg	50 Meg	.750 ±.060 (19.05 ±1.52)	.140 ±.030 (3.56 ±.76)	.025 ±.002 (.64 ±.05)

SUMMARY

The Rogowski coil due to the given task was designed and assembled. It was concluded that the inner dielectric of two popular cables RG-58 and RG-59 is suitable as core for the coil. The maximal output voltage of the coil was chosen approximately 3 kV. With this fact the number of the coil turns in RG-58 case is 88 and 56 for RG-59. Because first cable was more available for author it was used for the coil building. The passive integration method with RC circuit was chosen. The precision rated correctly components therefore were found. In both cases the maximal output voltage of the integrator was close to 30 mV.



If an external interference or percussive excitation take place in practice opposite or bifilar winding of the coil is recommended respectively (see figure on the left).

REFERENCES :

V.V. Panin, B.M. Stepanov (book in Russian)
"Measurement of magnetic and electrical fields"
Moscow Engineering Physical Institute, 1982

M. Argüeso, G. Robles, J. Sanz
"Measurement of high frequency currents with a Rogowski coil"